

Research Paper

## **The Ineffectiveness of the Correlation Coefficient for Image Comparisons \***

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# The Ineffectiveness of the Correlation Coefficient for Image Comparisons

## **ABSTRACT**

Pearson's  $r$  linear correlation coefficient is widely used for comparing images. Image processing experts appear to be cognizant of its serious limitations. This information, however, has not been communicated well to non-experts (including those working on security applications), who sometimes use the correlation coefficient without being aware of its problems. We discuss the disadvantages of the correlation coefficient and show specific examples of strikingly poor performance in the context of security.

## INTRODUCTION

Pearson's correlation coefficient,  $r$ , is widely used in statistical analysis, pattern recognition, and image processing [1-8]. Applications for the latter include comparing two images for the purposes of image registration, object recognition, and disparity measurement. For monochrome digital images, the Pearson correlation coefficient is defined as [1,4,7]:

$$r = \frac{\sum_i (x_i - x_m)(y_i - y_m)}{\sqrt{\sum_i (x_i - x_m)^2} \sqrt{\sum_i (y_i - y_m)^2}}$$

where  $x_i$  is the intensity of the  $i$ th pixel in image 1,  $y_i$  is the intensity of the  $i$ th pixel in image 2,  $x_m$  is the mean intensity of image 1, and  $y_m$  is the mean intensity of image 2.

The correlation coefficient has the value  $r=1$  if the two images are absolutely identical,  $r=0$  if they are completely uncorrelated, and  $r=-1$  if they are completely anti-correlated, for example, if one image is the negative of the other.

The correlation coefficient is used for security applications such as surveillance, treaty verification, tamper detection using security seals, and tagging [9-14]. (Seals leave unerasable evidence of entry or tampering; tags are devices or procedures that uniquely fingerprint an object.) Typically, the correlation coefficient is used to compare two images of the same object (or scene), taken at different times [11-14]. The  $r$  value indicates whether the object has been altered or moved.

In theory, we would obtain an  $r$  value of 1 if the object is intact, and a value of less than 1 if alteration or movement has occurred. In practice, distortions in the imaging system, pixel noise, slight variations in the object's position relative to the camera, and other factors produce an  $r$  value less than 1, even if the object has not been moved or physically altered in any manner [13]. In our experience with security applications, typical  $r$  values for two digital images of the same scene, one recorded immediately after the other using the same imaging system and illumination, range from 0.95 to 0.98. This is the same range as that reported by Palm and DeVolpi [13] for scanning electron microscope images.

Usually there needs to be an empirical definition of a threshold  $r$  value that indicates a breach of security. In other words, it is necessary to determine the minimum  $r$  value needed to conclude with confidence that the image is unchanged. For several security applications for which we are familiar, the chosen threshold values for  $r$  range from 0.30 to 0.85, depending on the application [9-14].

The purpose of this paper is to offer simple examples of the poor performance of the correlation coefficient for image comparison, particularly with security applications in mind. We know of no similar presentation. The problems and limitations of the correlation coefficient have been discussed previously [2, 3, 13-17], but briefly and abstractly, without specific image examples. Furthermore, the knowledge that the correlation coefficient often performs poorly does not seem to have been communicated well to non-experts. We conducted a survey of 50 image processing texts, introductory through advanced, that discussed the correlation coefficient. Only 3 of the 50 texts discussed potential problems and limitations of the correlation coefficient, and then mostly in terms of its computational intensiveness, rather than performance problems.

## **STRENGTHS AND WEAKNESSES OF THE CORRELATION COEFFICIENT.**

One of the obvious advantages of Pearson's correlation coefficient is that it condenses the comparison of 2 (often large) two-dimensional images down to a single scalar,  $r$ . Additionally, the correlation coefficient is completely invariant to linear transformations of  $x$  or  $y$  [1, 4, 13]. As a result,  $r$  is insensitive (within limits) to uniform variations in brightness or contrast across an image. Such spatially uniform variations can be caused, for example, by differences in the brightness of the light source over time, by changing background light levels, or by variations in the gain of the imaging system.

Despite its advantages, the correlation coefficient has many problems and limitations. The most widely recognized disadvantage is that it is computationally intensive. This often limits its usefulness for image registration (that is, orienting and positioning two images so they overlap). The correlation coefficient is also extremely sensitive to the image skewing, pincushioning, and vignetting that inevitably occur in imaging systems. Such distortions are particularly prevalent in scanning electron microscope pictures [13] because of the non-linearities and complexities of electron optics. (Image skewing occurs when an image slants in one direction. Pincushioning means that the edges of the image are concave. Vignetting is a reduction in the image intensity near the edges due to optical light collecting considerations.)

Another problem often overlooked in practical applications is that  $r$  is undefined--due to division by zero--if one of the test images has constant, uniform intensity. Users, especially for automated security applications, must be careful that the computer or micro-processor doing the  $r$  calculations does not permit an undefined value to default to  $r=0$  or  $r=1$ .

Other problems with the correlation coefficient include possible bias [16], complexities of interpretation [1], over-sensitivity to pixel noise and gain variations [2, 5, 15], difficulties in dealing

with perspective or with moving illumination sources [2,15, 17], undesirable behavior for images containing too much fine structure or too little [2, 15], and trouble in dealing with images having strong spatial disparity gradients [2].

There is an additional problem with the correlation coefficient that does not appear to have been discussed or demonstrated in detail elsewhere. An image can be greatly modified, without this being detected, as long as the local mean and/or histogram of pixel intensities are relatively unchanged. This is demonstrated below.

## **EXAMPLES OF POOR PERFORMANCE**

The top image in figure 1 shows an 8-bit (monochrome) 512 X 512 scanning electron microscope image of a metal surface. The complexity of the surface can be used as a unique identifier, or tag [13]. For the bottom image in figure 1, we have used a computer to artificially modify the top image such that the letters "LANL" replace a portion of the original image. The intensity for the letters was chosen such that the mean intensity of the region being overwritten did not change. Even though the human eye can immediately judge that the top and bottom images in figure 1 are dramatically different, the correlation coefficient reports that these two images are essentially identical:  $r=0.94$ . This is about the same correlation coefficient one gets by re-recording a second image of the surface shown in the top of figure 1.

Figure 2 shows an even more disturbing example of the failure of the correlation coefficient to detect changes in an image. Recognizing that individual pieces are missing is critical for applications such as reflective particle tags (RPTs). These consist of small, highly reflective particles attached to an object. RPTs uniquely fingerprint the object based on their complex spatial distribution [12].

The top image in figure 2 is a monochrome 8-bit video image with 512 X 512 pixels. The image shows a collection of plastic-coated paper clips spread out randomly on a surface. The bottom image in figure 2 shows another image of the scene except that one of the paper clips has been physically removed prior to recording this second image. The dark paper clip missing from the bottom image in figure 2 can be seen slightly up and to the right of center in the top image. The correlation coefficient for the top vs bottom image in figure 2 is  $r=0.98$ .

Now one way to improve the performance of the correlation coefficient--and make it less sensitive to image skewing, pincushioning, vignetting, or imperfect registration--is to compute  $r$  over a subset region of the complete image [13]. Even this approach, however, can still produce poor performance. If only the region inside the rectangle in figure 3 is used to compute  $r$ , the value is still quite large:  $r=0.86$ . The correlation coefficient is thus still reporting an almost unchanged image, even though the missing paper clip is a major feature inside the (200 X 200 pixel) rectangle!

Finally, in figure 4, the correlation coefficient fails to be useful for Elvis sightings. The two images are clearly different, yet  $r = 0.94$ . The bottom image was created by a superposition of the top image and an image of Elvis, such that the resulting local mean and histogram of pixel intensities were largely unchanged.

## **CONCLUDING REMARKS**

In summary, the correlation coefficient often fails to find differences in images that are widely disparate. In the case of a security system utilizing the correlation coefficient, an adversary can modify an object or scene quite dramatically and yet still go undetected, especially if he approximately preserves the local intensity mean and/or intensity histogram. Of particular significance for security applications, the correlation coefficient often fails to detect missing objects

within an image. Performance often improves only modestly if the correlation coefficient is computed for subset windows of the entire image.

Even when the correlation coefficient does perform acceptably, there are usually better algorithms for image comparison. Typically, the optimum choice of algorithms depends critically on general characteristics of the relevant images, and details of the application. One fact often overlooked is that the use of human vision with a blink comparator [10, 18] can often dramatically outperform even very sophisticated computer algorithms.

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## FIGURES

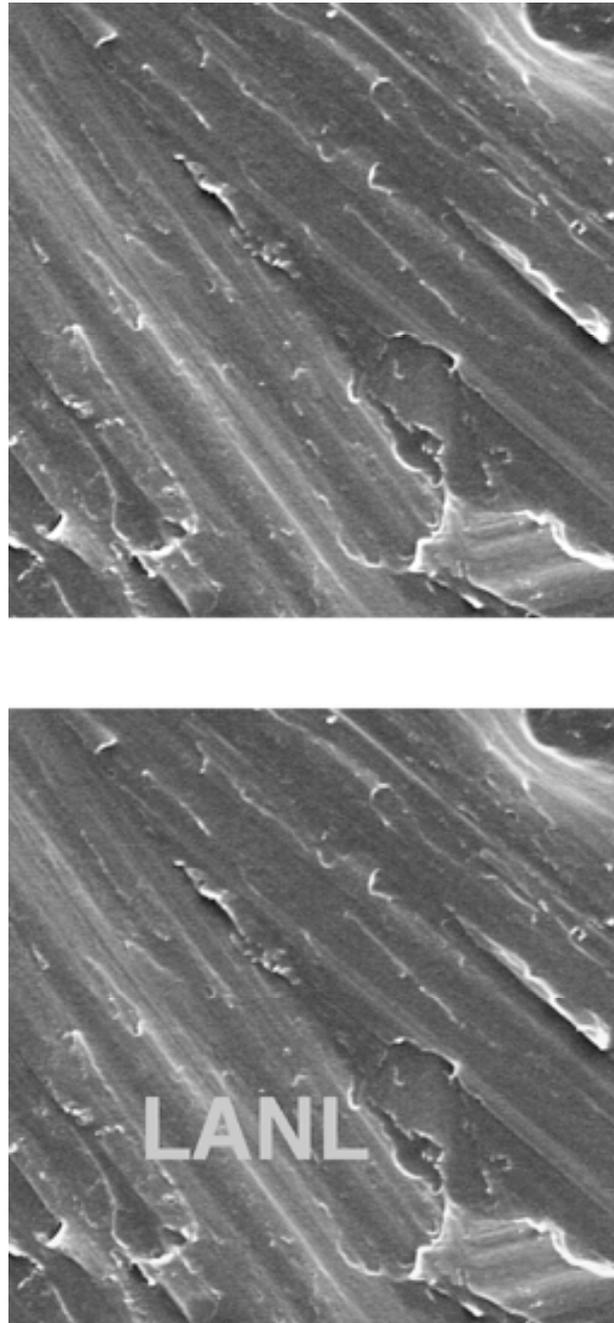


Figure 1. Comparing two images using the correlation coefficient. Both images are 8-bit, 512 X 512 scanning electron microscope images of a metal surface. The image at the bottom is the same as at the top except that the letters "LANL" have been overwritten. The pixel intensity of the letters is approximately equal to the local mean intensity of the original (top) image. The correlation coefficient for the top image vs. the bottom image is  $r=0.94$ . The correlation coefficient reports little difference.

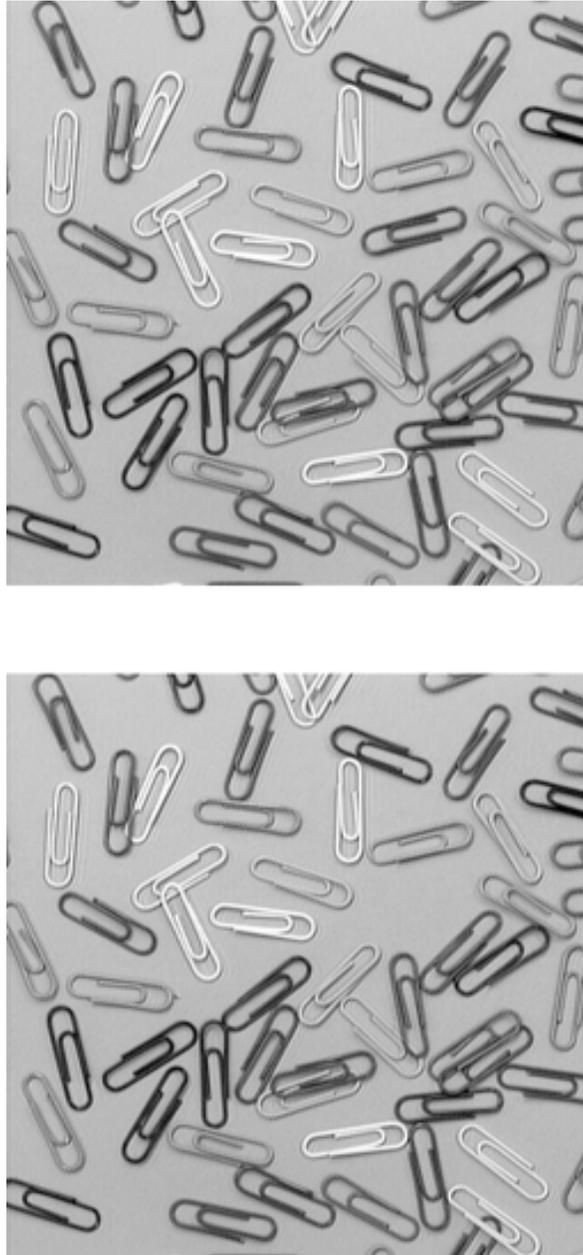


Figure 2. The correlation coefficient can be relatively poor at detecting missing objects in an image. Both the top and bottom images are 512 X 512 8-bit video images of plastic-coated paper clips. For the bottom image, one of the paper clips has been picked up and the image re-acquired. The missing paper clip is not detected: the correlation coefficient for the top image vs. the bottom image is  $r=0.98$ .

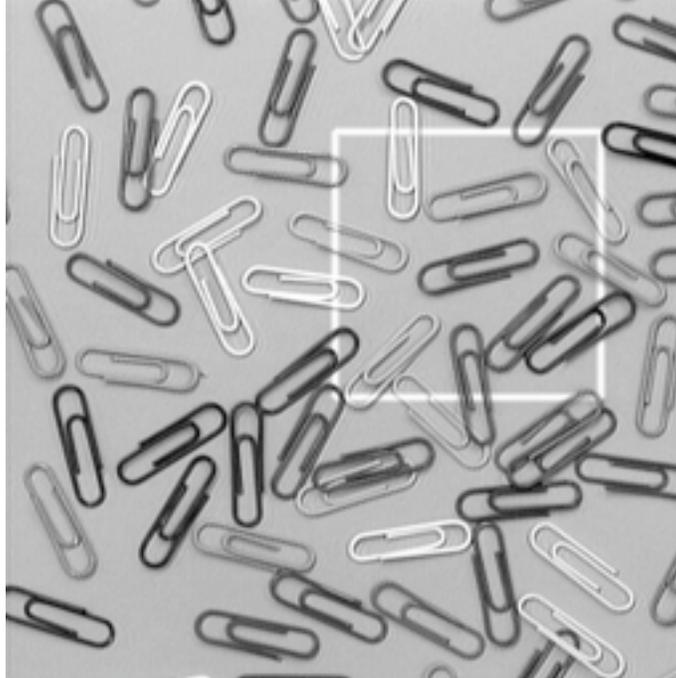


Figure 3. Reducing the window size only modestly helps the correlation coefficient detect the missing paper clip. If the correlation coefficient is recomputed for the top and bottom images in figure 2, limited to the subset (200 X 200) rectangle shown in this figure, then  $r=0.86$ .

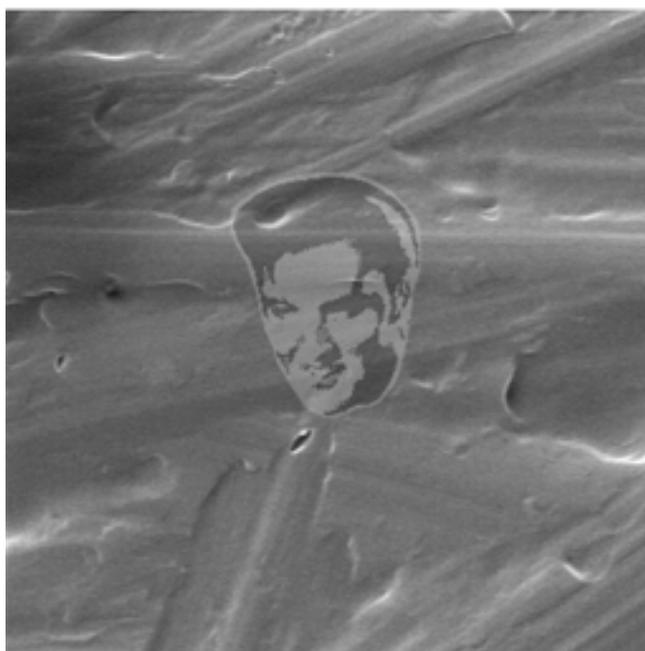
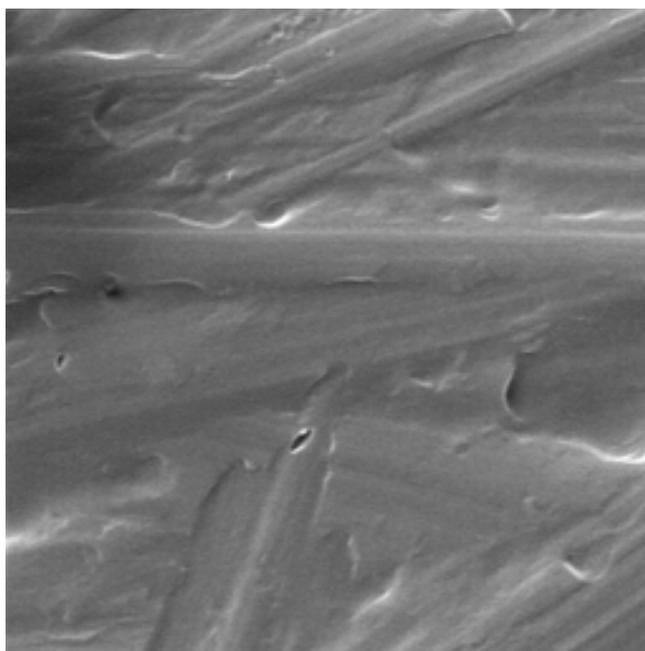


Figure 4. Scanning electron microscope images. The correlation coefficient fails to definitively detect the ghostly appearance of Elvis in the bottom image. For the top image vs. the bottom image,  $r=0.94$ .